Computer Algorithm to Calculate Longshore Energy Flux and Wave Direction from a Two Pressure Sensor Array

by
Todd L. Walton, Jr. and Robert G. Dean

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<td>A documented (FORTRAN IV) computer program is discussed as originally written for the CERC Longshore Sand Transport Research Program to analyze wave data collected at Channel Islands Harbor, California.</td>
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The program performs the basic analysis of two wave gage pressure records necessary to compute wave direction and wave energy at a given frequency and (continued)
computes the longshore energy flux used in sand transport for the entire energy spectrum of the wave record. This program uses linear wave theory for the wave transformation process and includes the assumption of straight and parallel bottom contours necessary for application of Snell's law of refraction.

The necessary steps in an analysis of wave data and sample outputs for some wave records from the Channel Islands wave gage pressure sensor pair are given. The program presently accepts data in the standard CERC magnetic-tape format where record lengths consist of 4,100 values.
This report provides coastal engineers with documentation necessary to compute the longshore energy flux used in sand transport rate calculation when random waves are present and synchronous data from two closely spaced pressure transducers exist. The documentation is based on a 3-year data collection effort and study of sand transport rates at Channel Islands Harbor, California. The computer program documented herein was used in wave data analysis for a two pressure sensor array installed in 30 feet of water at the site. The work was carried out under the U.S. Army Coastal Engineering Research Center's (CERC) Littoral Data Collection work unit, Shore Protection and Restoration Program, Coastal Engineering Area of Civil Works Research and Development.

This report was prepared by Dr. Todd L. Walton, Jr., Hydraulic Engineer, CERC, and Dr. Robert G. Dean, Department of Civil Engineering and College of Marine Studies, University of Delaware. Dr. Walton worked on the project under the general supervision of Dr. J.R. Weggel, Chief, Evaluation Branch, and Mr. N. Parker, Chief, Engineering Development Division.

Technical Director of CERC was Dr. Robert W. Whalin, P.E., upon publication of this report.

Comments on this publication are invited.

Approved for publication in accordance with Public Law 166, 79th Congress, approved 31 July 1945, as supplemented by Public Law 172, 88th Congress, approved 7 November 1963.

TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director
U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

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$^1$ To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9)(F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9)(F - 32) + 273.15$. 

1
**SYMBOLS AND DEFINITIONS**

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<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$a_1, b_1$</td>
<td>Fourier series coefficients</td>
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<tr>
<td>$B$</td>
<td>distance from bottom to pressure sensors</td>
</tr>
<tr>
<td>$C_g$</td>
<td>wave celerity</td>
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<td>$C_{12}$</td>
<td>cospectrum value</td>
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<td>$E$</td>
<td>wave energy density</td>
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<td>$F$</td>
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<td>$f_n$</td>
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<tr>
<td>$G_{B}, G_{BP}$</td>
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<td>$g$</td>
<td>acceleration of gravity</td>
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<td>$H_b$</td>
<td>breaking wave height</td>
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<td>index to account for gage number</td>
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<td>frequency number, argument of Fourier series coefficients</td>
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<td>$P_{ls}$</td>
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<td>average mean depth of water overlaying pressure sensors</td>
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<td>$\Delta f$</td>
<td>frequency step</td>
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<tr>
<td>$\Delta t$</td>
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<td>$\omega$</td>
<td>angular wave frequency</td>
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COMPUTER ALGORITHM TO CALCULATE LONGSHORE ENERGY FLUX AND WAVE DIRECTION FROM A TWO PRESSURE SENSOR ARRAY

by

Todd L. Walton, Jr. and Robert G. Dean

I. INTRODUCTION

The documented (FORTRAN IV programming language) computer program discussed in this report was originally written as part of the Coastal Engineering Research Center's (CERC) Longshore Sand Transport Research Program and was used in analysis of wave data collected at Channel Islands Harbor in conjunction with a study of sand transport at Channel Islands Harbor as discussed in Bruno, et al. (1981).

The program performs the basic analysis of two wave gage pressure records necessary to compute wave direction and wave energy at a given frequency and computes the longshore energy flux used in sand transport for the entire energy spectrum of the wave record. This program uses linear wave theory for the wave transformation process and includes the assumption of straight and parallel bottom contours necessary for application of Snell's law of refraction.

Necessary steps in the analysis of the wave data are presented in Sections II and III of this report. Subroutines are discussed and sample outputs for some wave records from the Channel Islands wave gage pressure sensor pair are given.

The program presently accepts data in the standard CERC magnetic-tape format where record lengths consist of 4,100 values. The first four values are the gage number and the date-time group, and the remaining 4,096 values are the pressures recorded in thousandths of a foot (head) of water at 0.25-second intervals. Should other input data be available, the program could easily be modified to accept the data by simple changes in the main program and in subroutines BUF and SWITCH.

Sample outputs have been presented for real wave data; some wave directional information cannot be obtained for all frequencies because the spectral information at some frequencies is ill-conditioned. The percent of energy for which this problem occurs is a small part of the energy (usually <3 percent) of the entire spectrum and is insignificant in energy-flux computations. Reasons for this feature are discussed later.

II. METHODOLOGY

Calculating the longshore energy flux at breaking required the following steps:

(1) Calculation of the frequency-by-frequency wave direction and energy at the location of the wave gages;

(2) determination of the breaking wave depth;

(3) transformation of the wave spectrum to the "breaker" line, including shoaling and refraction effects; and

(4) computation of $P_{ls}$, the longshore energy flux at the surfline.

Each of the steps is described below.

As noted previously, each of the input time-series pressure records consists of 4,096 data points with a time increment of 0.25 second. To reduce computational costs, modified time series are formed for analysis by averaging four adjacent data points. These new time series contain 1,024 data points spaced at 1.0-second intervals. This increases the aliasing period from 0.5 to 2.0 seconds; however, this is justified as the pressure response factor for a water depth of 6 meters and a wave period of 2 seconds is approximately 0.005.

The time series are analyzed using a standard fast Fourier transform (FFT) program to determine the coefficients. For example, for pressure time series from gage 1

\[
P_1(j) = \sum_{n=0}^{N-1} \left[ a_1(n) - ib_1(n) \right] \exp\left( \frac{i2\pi nj}{N} \right)
\]  

(1)

in which \( i = \sqrt{-1} \) and \( N \) is the total number of data points, \( T/\Delta t = 1,024 \), where \( T \) is the time series record length of 1,024 seconds, \( \Delta t \) the time increment of 1 second between samples, and \( j \) a discrete time \( t_j \) where \( t_j = \) discrete time value \( = j\Delta t \). The FFT coefficients are defined in terms of the pressure time series as

\[
a_1(n) - ib_1(n) = \frac{1}{N} \sum_{j=0}^{N-1} P_1(j) \exp\left( -i \frac{2\pi nj}{N} \right)
\]  

(2)

where the argument "n" of the Fourier coefficients \( a(n) \) and \( b(n) \) specifies the quantity to be a discrete function of wave frequency, \( f_n \), where \( f_n \), a discrete frequency value, is \( n\Delta f \) (where \( \Delta f = 1/T \)) and the \( a_1(0) \) term represents the mean value of the time-series pressure record for wave gage 1. Similar relationships exist for wave gage 2. In calculating the FFT coefficients, there are several options that may be employed in an attempt to reduce spectral leakage which arises due to representing an aperiodic time series by a periodic series. A large number of possible data windows (weighting functions for data) have been developed to reduce the adverse effects of spectral leakage (Harris, 1974). These can be expressed in the form of a weighting function \( w(j) \), such that the modified time series \( p'(j) \) is of the form

\[
p'(j) = w(j) p(j)
\]

in which \( p(j) \) is the digitized measured pressure value at time \( t_j = j\Delta t \), and \( w(j) \) a weighting function. A characteristic of these weighting functions is that they are equal to unity at the midpoint of the time series and decrease to a lesser value near the two ends. In the present program, a "cosine bell" weighting function is used; however, through comparisons of \( P_{gs} \) with and without this function, it was established that the effect of the weighting function was minimal (<5 percent). The cosine bell weighting function is expressed by

\[
w(j) = \frac{1}{2} \left( 1.0 - \cos \frac{2\pi j}{N} \right)
\]  

(3)
It is clear that the application of a weighting function will reduce the total energy in the record. This effect is partly compensated for by the following equation:

\[
p''(j) = \frac{\sqrt{\langle p^2 \rangle}}{\langle p'^2 \rangle} p'(j) \tag{4}
\]

thereby ensuring the same total energy in the altered and original time series, where \( \langle p^2 \rangle \) is the mean square value of the original time series and \( \langle p'^2 \rangle \) the mean square value of the weighted time series. It is the altered time series \( p''(j) \) that is subjected to FFT analysis. The primes will be dropped hereafter for convenience. The average mean depth of water overlying the pressure sensors, \( \Delta d \), is obtained by averaging the \( m \) time series to obtain \( a_m(0) \). For two separate time series records, \( m = 1, 2 \) (wave gages 1 and 2),

\[
\Delta d = 0.5 \left( a_1(0) + a_2(0) \right) \tag{5}
\]

The total water depth, \( d \), is the sum of \( \Delta d \) and the distance, \( B \), of the pressure sensors above the bottom (in later examples \( B = 0.76 \) meter).

Each FFT pressure coefficient is transformed to a water surface displacement coefficient by the following linear wave theory relationship discussed in the Shore Protection Manual (SPM) (see Ch. 2, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977):

water surface coefficients \quad dynamic pressure coefficients

\[
\begin{bmatrix} a_m(n), b_m(n) \end{bmatrix}_\eta = \frac{1}{\gamma K_z(n)} \begin{bmatrix} a_m(n), b_m(n) \end{bmatrix}_p \tag{6}
\]

in which the subscripts \( \eta \) and \( p \) denote water surface and dynamic pressure coefficients, respectively. The factor

\[
K_z(n) = \frac{\cosh k(n) B}{\cosh k(n) d} \tag{7}
\]

where \( \gamma \) is the specific weight of fluid (seawater) and is included when pressure coefficients are in normal units of pressure (i.e., N/M² or equivalent). In equation (7), \( B \) represents the distance of the pressure sensors above the bottom and \( k(n) \) is the wave number associated with the angular frequency, \( \omega(n) = \left( 2\pi n \Delta f \right) \), as obtained from the linear wave theory dispersion relationship

\[
\omega(n)^2 = g k(n) \tanh k(n) d \tag{8}
\]

One of the disadvantages of measuring waves with near-bottom pressure sensors is evident by examining equations (6) and (7). For the higher frequencies (shorter wave periods) \( K_z(n) \) is very small which means that the higher frequency waves result in very small pressure fluctuations near the sea floor. Thus, to avoid contaminating the calculated water surface displacements, it is
usually necessary to apply a high frequency cutoff, above which the pressure contributions are discarded. The proper selection of this high frequency cutoff depends on the signal to noise characteristics of the pressure sensor and the signal conditioning system. In the present program, the high frequency cutoff was established at a wave period of 3.0 seconds. Wave gage analyses by Thompson (1980) have shown that a 3.0-second high frequency spectral cutoff value provides reasonable estimates of total wave energy at west coast (U.S.) locations.

Denoting hereafter the FFT coefficients for the water surface as \(a(n)\) and \(b(n)\), it is noted that the coefficients have the following properties:

\[
\langle \eta^2 \rangle = \sum_{n=1}^{N-1} [a^2(n) + b^2(n)]
\]

and

\[
a \left( \frac{N}{2} + n \right) = a \left( \frac{N}{2} - n \right)
\]

\[
b \left( \frac{N}{2} + n \right) = -b \left( \frac{N}{2} - n \right)
\]

and thus

\[
\langle \eta^2 \rangle = 2 \sum_{n=1}^{N/2} [a^2(n) + b^2(n)]
\]

Thus, the total (kinetic and potential) energy \(E(n)\) associated with a particular wave frequency component, \(n\), is

\[
E(n) = 2\gamma [a^2(n) + b^2(n)]
\]

Now consider two wave or pressure sensors located at \((x_1, y_1)\) and \((x_2, y_2)\) (see Fig. 1). The results will be developed considering discrete frequencies.

Figure 1. Definition sketch for two sensor array.
The water surface displacement consistent with the assumption of one direction per frequency is

\[ \eta(x, y, j) = \sum_{n=0}^{N-1} \mathcal{F}(n) \exp \{i[n \omega_1 t - k_x(n) x - k_y(n) y]\} \]

\[ = \sum_{n=0}^{N-1} [a(n) - ib(n)] \exp \left( \frac{i2\pi nj}{N} \right) \]

(14)

where \( \omega_1 \) is the primary analysis frequency (\( = 2\pi/\text{record length} = 2\pi/T = 2\pi \Delta f \)), and \( \Theta(n) \) the direction of wave propagation at frequency \( \omega(n) = n \omega_1 \). The wave number components, \( k_x(n) \) and \( k_y(n) \), are expressed in terms of the wave number, \( k(n) \), and wave direction, \( \Theta(n) \), as

\[ k_x(n) = k(n) \cos \Theta(n) \]

\[ k_y(n) = k(n) \sin \Theta(n) \]

(15)

(16)

The cross spectrum, \( S_{12}(n) \), of the two measured water surface displacements (or dynamic pressures) is given by

\[ S_{12}(n) = |\mathcal{F}(n)|^2 \left\{ \exp - i \left[ k(n) \cos \Theta(n)(x_2 - x_1) + k(n) \sin \Theta(n)(y_2 - y_1) \right] \right\} \]

(17)

Denoting the separation distance and angle as \( \lambda \) and \( \beta \), respectively, the cross spectrum can be expressed as (see Fig. 1)

\[ S_{12}(n) = |\mathcal{F}(n)|^2 \left\{ \cos \left[ k(n) \lambda \cos (\Theta(n) - \beta) \right] - i \sin \left[ k(n) \lambda \cos (\Theta(n) - \beta) \right] \right\} \]

\[ = \text{cospectrum (n)} - i \quad \text{quad-spectrum (n)} \]

(18)

Thus, from equation (18), the wave direction \( \Theta(n) \) associated with each wave frequency can be expressed as

\[ \Theta(n) = \beta \pm \cos^{-1} \left\{ \frac{1}{k(n) \lambda} \tan^{-1} \left[ \frac{Q_{12}(n)}{C_{12}(n)} \right] \right\} \]

(19)

The above relationship has two roots, one of which must be selected based on physical considerations of the most likely direction of wave propagation. In the present case, assuming no wave reflection from the beach, the ambiguity in wave direction is ruled out; for wave sensors nearly parallel to the beach, the minus sign in equation (19) is appropriate.
There are two conditions for which it was not possible to calculate the wave directions $\Theta(n)$. These include poorly conditioned wave data, presumably due to spectral leakage, and spatial aliasing due to large separation distance between the two gages. If the data are poorly conditioned for determining wave direction, the absolute value of the quantity within the brackets \(-\) in equation (19) may exceed unity, a physically impossible condition since the extreme values of the cosine function are \(\pm 1\). This tends to occur for the extremely long waves for which the energy is small and the value of $k(n)$ is also small, the latter tending to result in large values of the bracketed quantity. The percentage of energy for which this condition occurred in the analysis of one year's wave data collected at Channel Islands Harbor was relatively small, averaging 2 to 3 percent with a maximum of approximately 10 percent. The second condition is related to spatial aliasing and requires that one-half the wavelength be equal to or greater than the projection of the wave gage separation distance in the direction of wave propagation. Referring to Figure 1,

$$L > 2L \{\cos[\Theta(n) - \beta]\}_{\text{max}}$$

which indicates that for the least adverse effects of spatial aliasing, the gages should be on an alinement parallel to the dominant orientation of the wave crests. As will be discussed later, in calculating $P_{\text{ls}}$ an attempt was made to account for this effect of aliasing by augmenting the calculated values, illustrated as follows by

$$(P_{\text{ls}})_{\text{cm}} = (P_{\text{ls}})_c \frac{E_{\text{TOT}}}{E}$$

in which the subscripts $c$ and $cm$ indicate calculated and calculated modified, respectively. $E_{\text{TOT}}$ and $E$ represent the total wave energy values and the wave energy not affected by spatial aliasing or poorly conditioned wave data, respectively. The total wave energy is that energy in the wave spectrum below the high frequency spectral cutoff value.

2. **Transformation of Wave Spectrum to Breaker Line.**

At this stage, the wave energy and wave direction in the vicinity of the gages are determined. These values are then transformed to the breaker line accounting for wave refraction and shoaling.

To determine the wave breaking depth, the onshore-directed energy flux is calculated in accordance with the expression (based on Snell's law of refraction) and equated to an equivalent expressed in terms of wave characteristics at breaking.

$$\text{Onshore energy flux} = \sum_{n=1}^{N/2} \gamma_2 [a(n)^2 + b(n)^2] C_g(n) \cos \Theta(n)$$

$$= \frac{\gamma E_b^2}{8} C_{gb} \cos \Theta_b$$

(22)
Assuming that the breaking wave angle, $\Theta_b$, is small, that the waves will break under shallow-water conditions, and that the ratio of breaking wave height to depth is a constant, the breaking wave height, $H_b$, is then given by

$$H_b = \left\{ \sum_{n=1}^{N/2} 16 \left[ a(n)^2 + b(n)^2 \right] C_g(n) \cos \theta(n) \right\}^{0.4} \left( \frac{GB}{g} \right)^{0.2}$$

(23)

where GB is the ratio of root-mean-square (rms) breaking wave height to breaking depth, $GB = H_b/d_b$ (here assumed $GB = 0.78$). With the breaking depth known, each wave component is transformed to shore accounting for both wave refraction and shoaling based on linear wave theory.

Wave refraction is in accordance with Snell's law and the assumption that straight and parallel contours existed between the gage and breaking locations

$$\theta_b(n) = \sin^{-1} \left[ \frac{C_b}{C_R(n)} \right] \sin \theta_R(n)$$

(24)

where $C$ is linear wave celerity (see the SPM, Ch. 2) in which the $r$ subscripts denote the "reference (gage)" location.

With the wave energy and direction now known at the breaker line, the value of the longshore energy flux, $(P_{ls})_{cm}$, is readily determined

$$(P_{ls})_{cm} = R(P_{ls})_c$$

$$= R \left\{ 2\gamma \sum_{n=1}^{N/2} \left[ a^2(n) + b^2(n) \right] C_g(n) \left[ \cos \theta(n) \sin \theta(n) \right] \right\}$$

(25)

in which the factor $R$ is given by the ratio

$$R = \frac{E_{TOT}}{E}$$

as defined in and discussed in relation to equation (21).

III. MAIN PROGRAM DOCUMENTATION

The detailed programming steps in analysis for the longshore energy flux, $(P_{ls})_{cm}$, (which in this program is calculated in terms of rms wave height) are presented in this section. Program steps are numbered to correspond to areas in the program listing where computations are carried out. A program listing with corresponding numbered steps follows the program documentation. Note that preceding text has used the indexes $j$ and $n$ for time and frequency, respectively, while the program which follows uses the index $I$ for both time and frequency. A listing of the main program is presented in Figure 2. Program steps are as follows and refer to numbered parts of main program listing:
PROGRAM SPECTRUM INPUT OUTPUT TAPE INPUT TAPE OUTPUT TAPE

OUTPUT ALGORITHM TO CALCULATE LONGSHORE ENERGY FLUX FACTOR AND WAVE DIAMETER FOR TWO PRESSURE SENSOR ARRAY

MAIN PROGRAM

PROGRAM IS PRESENTLY SET UP TO TAKE A TIME SERIES OF 1024 POINTS IN MAIN

DIMENSION C(512)
DIMENSION F1(1024), F11(1024), F2(1024), F21(1024)
DIMENSION SIGMA(512), THETA(512), T1(512), T2(512)
DIMENSION W(1024)
DIMENSION CG(512)*8

FUNCTION(1024,F8.2)

LOGICAL END DATA END/.FALSE./

DEFINITIONS-FIXED VARIABLES

N=EXPERIMENTAL POWER DEFINING NUMBER OF TIME SERIES POINTS=1024
S=SPACING BETWEEN WAVE GAGES (FEET)
T=TIME STEP BETWEEN WAVE GAGES (SECONDS)
BETA =ANGLE DIFFERENCE BETWEEN WAVE GAGE ALIGNMENT AND SHORELINE (RADIANS)
SLOPE=SLOPE OF REACH AT POINT OF WAVE BREAKING
GAMMA=SPECIFIC HEAT OF FLUID (LBS/FT**3)
DISTANCE OF PRESSURE SENSORS ABOVE BOTTOM (FEET)
G=ACCELERATION OF GRAVITY (FEET/SECONDS)
GBP=RATION BREAKING WAVE HEIGHT/DEPTH FOR LINEAR THEORY COMPUTATION OF
WAVE HEIGHT
GBP=RATIO BREAKING WAVE HEIGHT/DEPTH FOR LINEAR THEORY COMPUTATION OF
WATER DEPTH GIVEN BREAKING WAVE HEIGHT

DEFINITIONS-FLOATING VARIABLES

AVG1=AVERAGE OF TIME SERIES 1
AVG2=AVERAGE OF TIME SERIES 2
C1=MEAN WAVE CELEBRITY
C2=X
CG1=X
CG2=X
CH=MEAN BREAKING WAVE CELEBRITY

C=N POINT TIME SERIES BEFORE AVERAGING

DEP=DEPTH OF WATER AT GAGE SITE FROM AVERAGES OF GAGES 1 AND 2
F11(1)=UNDEFINED/COMPLEX IMAGINARY PORTION OF TRANSFORM
F11(1)=TIME SERIES DATA GAGE1/COMPLEX REAL PORTION OF TRANSFORM
F21(1)=UNDEFINED/COMPLEX IMAGINARY PORTION OF TRANSFORM

F2M(1)=TIME SERIES DATA GAGE2/COMPLEX REAL PORTION OF TRANSFORM
FMOD(1)=TIME SERIES AMPLITUDE MODULUS SQUARED
MBR=MEAN BREAKING WAVE HEIGHT
IA(I)=5000 POINT DATA GROUP AND TIME SERIES RECORD
PNEGATIVE CONTRIBUTION TO LONGSHORE ENERGY FLUX FACTOR
PLANH=LONGSHORE ENERGY FLUX FACTOR
PNHELPOSITIVE CONTRIBUTION TO LONGSHORE ENERGY FLUX FACTOR
WI2(I)=WAVE SPECTRA OF SERIES 1=2

F=SCALING FACTOR FOR SCALING UP ENERGY OF NONUSABLE

P=PORTIONS OF DIRECTIONAL SPECTRA

N=MEAN WAVE CELEBRITY TO WAVE CELEBRITY
N=MEAN PERCENT OF ENERGY BEYOND SPACIAL ALIASING FREQUENCY

H=PERCENT OF ENERGY BELONGING LOW FREQUENCY CUTOFF
HSUM=PERCENT OF INCOHERENT ENERGY
SHF=SUM OF ENERGY WITH FREQUENCIES ABOVE SPACIAL

ALIASING FREQUENCY CUTOFF

SHG=SUM OF ENERGY FLUX

S=MEAN PERCENT OF ONSHORE ENERGY FLUX

SIGMA(I)=RADIAL FREQUENCY

BSUM=SUM OF ENERGY WITH FREQUENCIES BELOW LOW FREQUENCY CUTOFF
SUD=SUM OF ENERGY WITH FREQUENCIES HAVING INCOHERENT WAVE DIRECTION
SUM=SUM OF ENERGY
SUM=SUM OF SQUARES OF TIME SERIES 1 WITHOUT AVERAGE
SUM=SUM OF SQUARES OF TIME SERIES 2 WITHOUT AVERAGE
SUM=SUM OF SQUARES OF TIME SERIES 1 WITH AVERAGE
SUM=SUM OF SQUARES OF TIME SERIES 2 WITH AVERAGE

T=TIME PERIOD

THETA(I)=WAVE DIRECTION IN RADIANS

Figure 2. Listing of main program.
C C TABlE BREAKING WAVE ANGLE C C TSMUM OF SQUARES OF DATA WINDOW MODIFIED TIME SERIES 1 31TUM1=14139265 C C TSMUM OF SQUARES OF DATA WINDOW MODIFIED TIME SERIES 2 TSWUM1=26119431

80 N=2.5*K 31±0,0 DELT=1.00 S=1.0,

90 8=E1=1.5708 SLPM=0.05 GAMMA=64.0

C C HIGH FREQ CUTOFF NO SEC SPACIAL ALIASING CUTOFF NO SEC C C NLF=HIGH FREQUENCY CUTOFF NUMBER NF=HIGH FREQUENCY CUTOFF NUMBER C C NSAFR=SPACIAL ALIASING FREQUENCY CUTOFF NUMBER F=TIME SERIES LENGTH/CUTOFF PERIOD

100 NLF=50 NF=30 NF=0.01 NSAFR=301 NSAFR=1.0

105 110 CONTINUE

C C INITIALIZING VALUES C LFR=0.00 SLD=0.00 SUM=0.00 SUM=0.00 SUM=0.00 SUM=0.00

115 SUMF1=0.0 SUM=0.0 SUM=0.0 AV=0.0 AV=0.0

120 PLP=0.0 PLN=0.0 PLN=0.0 B=0.0 UU 24 I=1

125 FI(I)=0.0 F2(I)=0.0

29 CONTINUE DD 30 I=1

30 CONTINUE

C C C C THIS PORTION OF PROGRAM READS IN WAVE PRESSURE VALUES INTO F1R,F2R ARRAYS AND ASSURES MATCHING DATE GROUPS FOR DIRECTIONAL WAVE ANALYSIS OF TWO CALL BUF(MGAGE1,M0NTH1,MOY1,MTIME1,F1R,10000) CALL BUF(MGAGE2,M0NTH2,MOY2,MTIME2,F2R,10000) IF(ENO) GO TO 1 IF(ENO) GO TO 1 IF(ENO) GO TO 1 IF(ENO) GO TO 1 IF(ENO) GO TO 1 IF(ENO) GO TO 1

Figure 2. Listing of main program.—Continued
Figure 2. Listing of main program.—Continued
205  SUMF2=SUMF2+FSXI(I)**2.+FSXI(I)**2.
90  CONTINUE
    SUMF1=SUMF1+MEAN1**2.
    SUMF2=SUMF2+MEAN2**2.
    WHITE(0)+29
210  SUMF1=SUMF1+MEAN1**2.
    SUMF2=SUMF2+MEAN2**2.
    WHITE(0)+29
215  FOHMFAT(/,7X,ITA(I)+10X*(SIGMA(I)+11X+(FMOBG(I))
    CALL *VLEN(DEPTH1,THETA(I))
    CALL *VLEN(DEPTH1,THETA(I))
220  IF(CI2(I).LE.0.00000001) GO TO 95
225  IF(A>B).GT.1.0) GO TO 91
230  FMUUP/I*SIGMA(I)
78  CONTINUE
92  CONTINUE
235  FMUUSU(I)*FIR(I)**2.+FIR(I)**2.
    XM=XM+XM/DEPTH
    XM=XM+XM/DEPTH
240  FMUUSU(I)*FMODSQ(I)/(XK**2.)
    FMUUSU(I)*FMODSQ(I)*RAI1
    FMUUSU(I)*FMODSQ(I)*RAI1
245  CN
    SG2=SUMMORE ENERGY FLUX
    SG2=SUMMORE ENERGY FLUX
250  CONTINUE
255  IF(I,GE,NYFR) GO TO 99
94  CONTINUE
260  IF(I,GE,NYFR) GO TO 99
79  SHFREQ*SHFREQ*FMOBG(I)
78  CONTINUE
77  SHFREQ=SHFREQ*FMOBG(I)
76  CONTINUE
99  CONTINUE
999  CONTINUE
8HG2=SHG2*SHG2
75  CONTINUE
50  CONTINUE
49  WHITE(I+50)+I*(SIGMA(I)+PCT*(16X,in(THETA(I))
74  CONTINUE
351  CONTINUE
48  CONTINUE
44  CONTINUE

Figure 2. Listing of main program.—Continued
This portion of program modifies wave gage angles to breaking wave angles and computes longshore energy flux factors.

270 \( \text{HB} = (\text{BH} + 2/4) \times (\text{CH} + 1/2) \times (\text{G})^{2/3} \)
275 \( \text{C} = (\text{CH} / \text{G})^{2/3} \)
280 \( \text{CONTINUE} \)

Figure 2. Listing of main program.—Continued
(1) Input data for this program are in the form of digital magnetic-tape records of 4,100 values. The first 4 values of the records are the gage number, month, day, and time of the observations; the last 4,096 values are the time-series pressure values of the wave gage. In the present program the wave gage pressures are stored in thousandths of a foot (head) water at 0.25-second intervals. Subroutine BUF reads time-series data into array CNTL, where it is averaged to provide 1,024 time-series values of \( \Delta t = 1 \) second spacing. Units are also divided by 1,000 to convert values to feet (head) of water.

(2) The date groups of record 1 and record 2 are compared to ensure that times of records are simultaneous; if the times are not, the program searches the record file until this condition is met. The two records are then checked for proper sequence to ensure that gage 1 is analyzed first. Subroutine SWITCH switches arrays if they are not in proper order.

(3) Each of the two 1,024 value time series is then analyzed for average values which are printed out along with the average depth of water at each gage site. The average value of each of the time-series records is again averaged and is added to the height of the gages above the bottom to obtain the water depth:

\[
    \text{DEPTH} = \frac{\text{AVERAGE 1} + \text{AVERAGE 2}}{2} + B
\]

in which AVERAGE 1 is the average of time series 1 = \( a_1(0) \), AVERAGE 2 the average of time series 2 = \( a_2(0) \), and \( B \) the height of sensors above the bottom.

An option to apply a weighting function \( w(j) = W(I) \) in program] has been incorporated before the FFT subroutine is called. In this particular program a cosine bell weighting function has been incorporated. If the data window option is selected, the two time-series data records, which are read into FIR and F2R arrays, are multiplied by the following weighting function (cosine bell)

\[
    w(j) = \frac{1}{2} \left[ 1 - \cos \left( \frac{2\pi j}{N} \right) \right]
\]

where \( j \) is the time step number and \( N \) the number of data points in series. If no weighting function is desired in analysis set \( w(j) = 1.0 \), which is the "box car" weighting function.

As the cosine bell function reduces the total energy content of the waves, the final energy obtained from the FFT must be rescaled up to the proper value. This is accomplished by scaling up the time-series pressure values by the ratio

\[
    R = \frac{\text{Unwindowed energy}}{\text{Windowed energy}} = \sqrt{\frac{\langle p^2 \rangle}{\langle p'^2 \rangle}}
\]

as discussed in equation (4).
(4) Cospectra and quad-spectra of the gages are computed using the following relationships (note in computer program index, \( I \) is used for frequency counter, \( n \)):

\[
\text{Cospectra} = C_{12}(I) = F_{1R}(I)F_{2R}(I) + F_{1I}(I)F_{2I}(I)
\]

\[
\text{Quad-spectra} = Q_{12}(I) = F_{1R}(I)F_{2I}(I) - F_{2R}(I)F_{1I}(I)
\]

in which \( F_{1R} \) and \( F_{1I} \) are the real and imaginary parts of complex transforms of time series 1; \( F_{2R} \) and \( F_{2I} \) are the real and imaginary parts of complex transforms of time series 2.

(5) Wave angle is calculated in accordance with equation (19).

\[
\theta(n) = \theta = \frac{\beta}{\arccos \left( \frac{1}{k(n)\ell} \cdot \arctan \frac{Q_{12}(n)}{C_{12}(n)} \right)}
\]

where \( k(n) \) is the wave number calculated via linear wave theory, \( \ell \) the spacing of gages, and \( \beta \) the difference in alinement of gages and shoreline in Figure 1.

Due to energy leakage problems in spectra, impossible wave angles can result [wave angles with \((1/k(n)\ell \, \arctan \, Q_{12}(n)/C_{12}(n)) > 1.0 \) ]. When this happens, energy is lumped into a separate category for later scaling up of the longshore energy flux.

(6) The high frequency cutoff in this particular program has been set at 2.09 radians per second, which corresponds to a period of 3 seconds or \( \text{NYFR} = 342 \). This value can be reset in the main program by adjustment of \( \text{NYFR} \) where

\[
\text{NYFR} = \frac{N\Delta t}{T_{HF}}
\]

and \( N \) is the number of data points in time series, \( \Delta t \) the spacing in time of data points, and \( T_{HF} \) the high frequency cutoff period. The spatial aliasing frequency is computed in subroutine HFC.

Energy between the spatial aliasing frequency and the high frequency cutoff is put into a special category and used to scale up the final longshore energy flux.

(7) Each frequency contribution to the onshore energy flux is calculated for the gage site location as follows:

\[
\text{Onshore energy flux} = 2\gamma |F_n(n)|^2 C_g(n) \cos [\theta(n)]
\]
where

$$|F_\eta(n)| = \text{modulus of the complex amplitude spectra of wave elevation above mean surface at gage site}$$

$$C_g(n) = \text{group wave speed at gage site}$$

$$\theta(n) = \theta = \text{angle of wave direction (see Fig. 1)}$$

$$\gamma = \text{specific weight of seawater}$$

The onshore energy flux is then summed to obtain the total onshore energy flux. In the program, onshore energy flux/$\gamma = HG2$.

(8) Breaking wave height at the shoreline is determined from the mean square onshore energy flux via a linear theory wave transformation formula which can be simplified to

$$H_b = \left[\frac{N}{2} \sum_{n=1}^{N/2} 16|F_\eta(n)|^2 C_g(n) \cos \theta(n)\right]^{0.4} \left(\frac{GB}{g}\right)^{0.2}$$

where $GB$ is the wave height-to-water depth ratio at breaking and $g$ the acceleration of gravity.

The choice of $GB$ is up to the user although a value of $GB = 1.42$ has been found by Komar and Gaughan (1972) to best fit wave tank data for breaking wave heights for monochromatic waves. In the present program, $GB$ has been set equal to $0.78$ but can be readily changed.

The breaking wave water depth is calculated from the equation

$$\frac{H_b}{d_b} = GBP$$

where $d_b$ is the wave breaking water depth and GBP the ratio of wave height to water depth at breaking.

In this case a different value of the ratio of breaking wave height to water depth can be used in the program for obtaining the proper water depth. An assumed value of GBP = 0.78 from the solitary wave theory in the SPM is used.

Linear wave celerity is assumed and breaking wave celerity is estimated as

$$C_b = \left(g \frac{H_b}{GBP}\right)^{0.5}$$

The breaking wave height and celerity calculated in this approach apply to all frequencies.
(9) Frequency-by-frequency modification of wave angles is made assuming linear wave theory, Snell's law, and parallel bottom contours offshore. The breaking wave angle, $\Theta_b(n)$, is calculated from

$$\Theta_b(n) = \arcsin \left[ \frac{C_b(n) \sin \Theta_r(n)}{C_r} \right]$$

where the subscript $r$ refers to the reference gage location.

(10) Longshore energy flux is calculated for each frequency component (except the special cases discussed in Sec. II) using the equation

$$P_{\ell s}(n) = \gamma |F_\eta(n)|^2 C_{gb}(n) \sin 2\Theta_b(n)$$

and is summed up to obtain a net longshore energy flux.

(11) The value of the net longshore energy flux is multiplied by a factor $R$ which scales up the total energy in the spectrum (below the high frequency cutoff). The equation for scaling factor $R$ is

$$R = \frac{1}{(1 - RTOT)}$$

where $RTOT = RSODD + RSHFRQ$ when $RSODD$ is the percent of energy in low frequency bands for which impossible values of the cosine function are calculated, and $RSHFRQ$ is the percent of energy between spacial aliasing frequency and high frequency cutoff.

The final result of analysis of the two gage records for the net longshore energy flux $PLNET$ is printed out, as well as specific frequencies for which impossible directional results occur and frequencies at which more than 2.5 percent of the total wave energy is found.

IV. SUBROUTINE DOCUMENTATION

1. FFT Subroutine.

The sampled time function, $f(j)$, will be expressed as

$$f(j) = \sum_{n=0}^{N-1} F(n) \exp(i\omega_1 j\Delta t)$$
in which
\[ \omega_1 = \frac{2\pi}{\text{record length}} = \frac{2\pi}{T} = \frac{2\pi}{N\Delta t} \]

\[ t_j = j\Delta t = \text{a discrete time where } j \text{ is the integer time step} \]

\[ F(n) = a(n) - ib(n) \]
\[ a\left(\frac{N}{2} + n\right) = a\left(\frac{N}{2} - n\right) , n \neq 0 , \frac{N}{2} \]
\[ b\left(\frac{N}{2} + n\right) = -b\left(\frac{N}{2} - n\right) , n \neq 0 , \frac{N}{2} \]
\[ a(0) = \text{mean of sampled record} \]
\[ b(0) = b\left(\frac{N}{2}\right) = 0 \]

Because negative indexes are not readily handled by most FORTRAN compilers, the summation extends over the interval \( 0 \leq n \leq N - 1 \), rather than over the symmetric interval \( -N/2 \leq n \leq N/2 \). From the definition of the coefficients above, it is clear that the coefficients \( a(n) \) and \( b(n) \) for \( n > N/2 \) contain no additional information.

The inverse relationship completing the FFT pair is

\[ F(n) = \frac{1}{N} \sum_{j=1}^{N} f(j) \exp(-in\omega_1\Delta t) \]

As an enumeration of the complex FFT coefficients, suppose the series of 8 values is considered, \( N = 8 \). The coefficients would be

\[ F(0) = a(0) \]
\[ F(1) = a(1) - ib(1) , F(7) = a(7) - ib(7) = a(1) + ib(1) \]
\[ F(2) = a(2) - ib(2) , F(6) = a(6) - ib(6) = a(2) + ib(2) \]
\[ F(3) = a(3) = ib(3) , F(5) = a(5) - ib(5) = a(3) + ib(3) \]
\[ F(4) = a(4) \]

This pattern prevails for all sets of FFT coefficients, regardless of the value of \( N \). Both \( F(0) \) and \( F(N/2) \) are real and, as noted previously, the coefficients \( F(n) \) for \( n > N/2 \) really contain no additional information. The FFT subroutine used here requires that the number of data points, \( N \), provided be an integral power of 2, i.e.,

\[ N = 2^K \]

where \( K \) is an integer. Thus analyses of 512, 1,024, or 2,048 data points \( (K = 9, 10, 11) \) would be suitable with this subroutine.
The following two requirements are satisfied in the FFT subroutine.

(a) By operating on the sampled function, obtaining the \( F(n) \) coefficients and carrying out the inverse FFT \((FFT^{-1})\), the original time function is recovered. Schematically,

\[
f(j) \rightarrow \text{FFT} \rightarrow F(n) \rightarrow \text{FFT}^{-1} \rightarrow f(j)
\]

(b) The mean square of the sampled time function is equal to the sum of the squares of the moduli of the FFT coefficients, \( F(n) \), i.e.,

\[
\frac{1}{N} \sum_{j=1}^{N} [f(j)]^2 = \sum_{n=0}^{N-1} |F(n)|^2
\]

a. Calling Statement: SUBROUTINE FFT (FR, FI, K, ICO) (see Fig. 3). FR, FI = real and imaginary coefficients in

\[ F(n) = FR(n) - iFI(n) \]

\[ K = \text{power of two (i.e., } N = 2^K) \]

ICO = control whether FFT or \((FFT)^{-1}\) operation is desired if

\[
\begin{align*}
ICO &= 0 \rightarrow \text{FFT} \\
ICO &= 1 \rightarrow (FFT)^{-1}
\end{align*}
\]

When entering the subroutine, FR is the time sequence \( f(j) \) and FI is arbitrary. When exiting the subroutine, FR and FI are the real and imaginary parts of the complex transform, respectively; e.g., input is

\[ K = 5 \]

\[ ICO = 0 \]

\[
f(j) = 1.0 + 2.0 \cos \frac{2\pi(j\Delta t)}{32} + 3.0 \cos \frac{4\pi(j\Delta t)}{32} - 0.6 \sin \frac{2\pi(j\Delta t)}{32} - 1.4 \sin \frac{4\pi(j\Delta t)}{32}
\]
Figure 3. Listing of FFT subroutine.

b. Data Input to Program.

\[
\begin{array}{c|cccc}
\text{f(j) values at} & 6.000 & 5.080 & 3.750 & 2.184 \\
\text{\(\Delta t = 1\) second} & 0.590 & -0.829 & -1.900 & -2.506 \\
\text{intervals} & -2.600 & -2.215 & -1.451 & -0.465 \\
\text{(32 values)} & 0.562 & 1.445 & 2.034 & 2.229 \\
& 2.000 & 1.391 & 0.513 & -0.475 \\
& -1.390 & -2.054 & -2.322 & -2.109 \\
& -1.400 & -0.257 & 1.188 & 2.755 \\
& 4.238 & 5.438 & 6.189 & 6.386 \\
\text{FR =} & 6.000 & 5.080 & 3.750 & 2.184 \\
\text{(32 values)} & 0.590 & -0.829 & -1.900 & -2.506 \\
& -2.600 & -2.215 & -1.451 & -0.465 \\
\end{array}
\]
0.562  1.445  2.034  2.229
2.000  1.391  0.513  -0.475
-1.390 -2.054 -2.322 -2.109
-1.400 -0.257  1.188  2.755
  4.238  5.438  6.189  6.386

FI =
(32 values)
0.000  0.000  0.000  0.000
0.000  0.000  0.000  0.000
0.000  0.000  0.000  0.000
0.000  0.000  0.000  0.000
0.000  0.000  0.000  0.000
0.000  0.000  0.000  0.000
0.000  0.000  0.000  0.000
0.000  0.000  0.000  0.000

Output:  FFT (XR, XI, 5, 0).

a(n) coefficients
(32 values)
0.000  0.000  0.000  -0.000
-0.000 -0.000 -0.000 -0.000
-0.000 -0.000 -0.000 -0.000
-0.000 -0.000 -0.000 -0.000
-0.000  0.000  0.000  0.000
  0.000  0.000  1.500  1.000

b(n) coefficients
(32 values)
0.000  -0.300  -0.700  -0.000
-0.000 -0.000 -0.000 -0.000
-0.000 -0.000 -0.000 -0.000
  0.000  0.000  0.000  0.000
  0.000  0.000  0.000  0.000
  0.000  0.000  0.700  0.300

At (time step) = 1 second in above example.

2. HFC Subroutine.

This subroutine resets the spatial aliasing frequency cutoff to a higher frequency than would be the case for normal incidence of waves to gage pair. In the present version of this subroutine, it has been assumed that the maximum angle which the wave crests can make with the gage pair axis is 45°. The spatial aliasing criteria are expressed in Figure 1, where for proper resolution of wave direction the following criteria must be met

\[ k(n) \cos \left[ \theta(n) - \beta \right] < \frac{L}{2} \]

\[ k(n) \cos \left[ \theta(n) - \beta \right] < k(n) \frac{L}{2} \]
The proper spatial aliasing frequency to correspond with the spatial aliasing wave number cutoff is found from the normal wave dispersion relationship.

Calling Statement: HFC (DEPTH, S, DELTT, N, NSALFR) (see Fig. 4).

DEPTH = depth of water at gage site (from main program)
S = spacing of wave gage pair (from main program) (= \( \ell \) in text)
DELTIT = time-step increment between values in time series analyzed (from main program)
N = exponent of 2 describing number of time series values (from main program)
NSALFR = integer number for spatial aliasing frequency cutoff

3. SWITCH Subroutine.

This subroutine is set up to interchange time-series data arrays in the instance when gage 2 data are processed before gage 1 data (see Fig. 5). If the first gage record processed is not equal to the appropriate number of the gage, as specified in main program, data arrays of first and second gage records are interchanged.

4. WVLEN Subroutine.

This subroutine accepts wave period and water depth as input and calculates wave number as output via a Newton-Raphson iteration.
Calling Statement: WVLEN (DPT, PER, XKH) (see Fig. 6).

DPT = water depth (from main program)
PER = wave period (from main program)
XKH = wave number * water depth (calculated in subroutine)

WAVE LENGTH ITERATION SUBROUTINE--THIS SUBROUTINE CALCULATES WAVELENGTH VIA NEWTON-RAPHSON ITERATION USING PERIOD,WATER DEPTH INPUT

DPT=WAVE PERIOD
PER=WAVE DEPTH
XKH=WAVE NUMBER*WATER DEPTH

SUBROUTINE WVLEN(DPT,PER,XKH)
XKH=(231853/PER)**2*DPT/32.22
IF(XKH>6.3)Z=1+1

GO TO 9
2 XKH=SUNIT(XKH)
3 SM=SUM(XKH)
4 CM=SUM(XKH)**2
5 EP=XKH*XKH**2/CH
6 SLUPE=XKH/CM**2*8H/CH
7 VXKH=EPS/SLUPE
8 IF(ASS(DXKH/XKH)=0.0001)9999
9 XKH=XKH+DXKH
GO TO 2
9 CONTINUE
RETURN
END

Figure 6. Listing of WVLEN subroutine.

5. BUF Subroutine.

This subroutine is set up to read in wave gage files from magnetic tape. The data records consist of arrays of 4,100 values, the first four of which are the gage number, month, day, and time of wave record. The remaining 4,096 values represent pressures in thousandths of a foot (head) of water. The data are returned to main program as a wave gage number-date series and a time series of 4,096 values of pressure in feet (head) of water. Two records are processed in one pass.

Calling Statement: BUF (MGAGE, MONTH, MDAY, MTIME, CNTL, IDATE, END) (see Fig. 7).

MGAGE = number of gage (read from tape)
MONTH = month of observation (read from tape)
MDAY = day
MTIME = time

CNTL = control array of 4,096 pressure values in feet (head) of water returned to main program

IDATE = summed time group for time comparison between gages
END = logical end
SUBROUTINE HUF READS IN WAVE GAGE DATE DATA AND TIME SERIES DYNAMIC PRESSURE VALUES IN FEET HEAD OF WATER THIS SUBROUTINE READS 4096 TIME SERIES VALUES AND AVERAGES TO OBTAIN 1024 VALUES FOR MAIN PROGRAM ANALYSIS MAGE=GAGE NUMBER MUNTH=MUNTH OF RECORDING MDAY=MAY OF RECORDING MTIME=TIME OF RECORDING HEAL=ARRAY OF AVERAGED TIME SERIES VALUES SUBROUTINE HUF(MGAGE, MUNTH, MDAY, MTIME, REAL, IDATE, END) DIMENSION CNTL(4096), IA(5000) DIMENSION REAL(1024) LOGICAL END

CONTINUE BUFFER IN(9,1)(IA(1):IA(5000)) IF CUNIT(9) 10,20,10 PRINT(11)(IA(1),I=1,A) CONTINUE MGAGE=IA(1) MUNTH=IA(2) MDAY=IA(3) MTIME=IA(4) IDATE=IA(2)+IA(1)*IA(3)+IA(4) NR=UA 25 IA1=0.0 CONTINUE 104 FORMAT(13) (PARITY ERROR ON +I1) CONTINUE MGAGE=IA(1) MUNTH=IA(2) MDAY=IA(3) MTIME=IA(4) IDATE=IA(2)+IA(3)+IA(4) NR=UA 25 IA1=0.0 CONTINUE CNTL(J)=IA(K) 25 CNTL(J)=CNTL(J)/1000. DD 26 UA=0.066+UA CNTL(J)=CNTL(4087) 35 UA=IA1 DD 27 L=IA1/1024 REAL(L)=(CNTL(J)+CNTL(J+1)+CNTL(J+2)+CNTL(J+3))/4. UA=IA1 27 CNTINUE RETURN END TRUE. RETURN END

Figure 7. Listing of BUF subroutine.

V. SAMPLE OUTPUT

Three examples of output are presented for different dates for the wave gage pair at Channel Islands Harbor (Fig. 8). The year the data was taken was 1975.

The first set of frequencies lists amplitude modules squared of wave data having impossible direction results. The sum total of this energy (in decimal percent) is listed as the quantity RSODD in the variable output at the bottom of the output. In the case of the wave data taken on 7-26-1600, the incoherent data amounted to 0.004 (0.4 percent) of the total energy in the wave record.

The second set of frequencies listed provides the wave direction for the frequency bands having a significant part of the energy (≥2.5 percent). In the case of the wave record taken on 7-26-1600, it is seen that the wave angle is reasonably consistent from the frequency-to-frequency band and is approximately 0.70 radian (40.1°).

The variable list provided at the bottom of the sampled output gives values of most importance in the analysis of wave information for longshore energy flux. The longshore energy flux output is in pounds per second units; the output in the first example is 89.23 pounds per second.
Example 1

<table>
<thead>
<tr>
<th>GAUGE NO.</th>
<th>MUNTH</th>
<th>DAY</th>
<th>TIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>311</td>
<td>7</td>
<td>26</td>
<td>1600</td>
</tr>
<tr>
<td>312</td>
<td>7</td>
<td>26</td>
<td>1600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>sigma(I)</th>
<th>FMDSQ(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.018408</td>
<td>0.000025</td>
</tr>
<tr>
<td>3</td>
<td>0.024544</td>
<td>0.000014</td>
</tr>
<tr>
<td>4</td>
<td>0.030680</td>
<td>0.000027</td>
</tr>
<tr>
<td>6</td>
<td>0.042951</td>
<td>0.000036</td>
</tr>
<tr>
<td>7</td>
<td>0.05223</td>
<td>0.000012</td>
</tr>
<tr>
<td>9</td>
<td>0.06359</td>
<td>0.000021</td>
</tr>
<tr>
<td>10</td>
<td>0.067495</td>
<td>0.000048</td>
</tr>
<tr>
<td>11</td>
<td>0.085903</td>
<td>0.000009</td>
</tr>
<tr>
<td>14</td>
<td>0.147262</td>
<td>0.000127</td>
</tr>
<tr>
<td>24</td>
<td>0.155398</td>
<td>0.000041</td>
</tr>
<tr>
<td>25</td>
<td>0.155670</td>
<td>0.000002</td>
</tr>
<tr>
<td>27</td>
<td>0.177942</td>
<td>0.000099</td>
</tr>
<tr>
<td>28</td>
<td>0.184078</td>
<td>0.000040</td>
</tr>
<tr>
<td>30</td>
<td>0.196350</td>
<td>0.000066</td>
</tr>
<tr>
<td>32</td>
<td>0.202485</td>
<td>0.000029</td>
</tr>
<tr>
<td>33</td>
<td>0.257709</td>
<td>0.000014</td>
</tr>
<tr>
<td>42</td>
<td>0.276117</td>
<td>0.000041</td>
</tr>
<tr>
<td>65</td>
<td>0.398835</td>
<td>0.000093</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>SIGMA(I)</th>
<th>PCT</th>
<th>THETA(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>67</td>
<td>0.111068</td>
<td>0.0390404</td>
<td>0.70274903</td>
</tr>
<tr>
<td>68</td>
<td>0.172427</td>
<td>0.04032986</td>
<td>0.77995085</td>
</tr>
<tr>
<td>73</td>
<td>0.4792239</td>
<td>0.0231251</td>
<td>0.88437628</td>
</tr>
<tr>
<td>74</td>
<td>0.5305931</td>
<td>0.1039194</td>
<td>0.69736113</td>
</tr>
<tr>
<td>75</td>
<td>0.6019424</td>
<td>0.06899780</td>
<td>0.69198090</td>
</tr>
<tr>
<td>78</td>
<td>0.7860201</td>
<td>0.02500115</td>
<td>0.7191254</td>
</tr>
<tr>
<td>79</td>
<td>0.8477993</td>
<td>0.0447282</td>
<td>0.63791396</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>SIGMA(I)</th>
<th>PCT</th>
<th>THETA(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>67</td>
<td>0.111068</td>
<td>0.0390404</td>
<td>0.70274903</td>
</tr>
<tr>
<td>68</td>
<td>0.172427</td>
<td>0.04032986</td>
<td>0.77995085</td>
</tr>
<tr>
<td>73</td>
<td>0.4792239</td>
<td>0.0231251</td>
<td>0.88437628</td>
</tr>
<tr>
<td>74</td>
<td>0.5305931</td>
<td>0.1039194</td>
<td>0.69736113</td>
</tr>
<tr>
<td>75</td>
<td>0.6019424</td>
<td>0.06899780</td>
<td>0.69198090</td>
</tr>
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<td>78</td>
<td>0.7860201</td>
<td>0.02500115</td>
<td>0.7191254</td>
</tr>
<tr>
<td>79</td>
<td>0.8477993</td>
<td>0.0447282</td>
<td>0.63791396</td>
</tr>
</tbody>
</table>

Figure 8. Three examples of output for wave gage pair at Channel Islands Harbor.
Example 2

<table>
<thead>
<tr>
<th>GAUGE NO.</th>
<th>MOUNT</th>
<th>DAY</th>
<th>TIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>311</td>
<td>7</td>
<td>26</td>
<td>1800</td>
</tr>
<tr>
<td>312</td>
<td>7</td>
<td>26</td>
<td>1800</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>( \Sigma(I) )</th>
<th>( \Phi_0(I) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00136</td>
<td>0.00169</td>
</tr>
<tr>
<td>2</td>
<td>0.01272</td>
<td>0.00099</td>
</tr>
<tr>
<td>3</td>
<td>0.01808</td>
<td>0.00007</td>
</tr>
<tr>
<td>5</td>
<td>0.03680</td>
<td>0.00013</td>
</tr>
<tr>
<td>8</td>
<td>0.04987</td>
<td>0.00066</td>
</tr>
<tr>
<td>9</td>
<td>0.05523</td>
<td>0.00114</td>
</tr>
<tr>
<td>10</td>
<td>0.06135</td>
<td>0.00009</td>
</tr>
<tr>
<td>11</td>
<td>0.067495</td>
<td>0.00000</td>
</tr>
<tr>
<td>13</td>
<td>0.079767</td>
<td>0.00168</td>
</tr>
<tr>
<td>14</td>
<td>0.08593</td>
<td>0.00020</td>
</tr>
<tr>
<td>15</td>
<td>0.092039</td>
<td>0.00013</td>
</tr>
<tr>
<td>16</td>
<td>0.098175</td>
<td>0.00012</td>
</tr>
<tr>
<td>18</td>
<td>0.10447</td>
<td>0.00005</td>
</tr>
<tr>
<td>19</td>
<td>0.108583</td>
<td>0.00006</td>
</tr>
<tr>
<td>23</td>
<td>0.14126</td>
<td>0.00004</td>
</tr>
<tr>
<td>25</td>
<td>0.159534</td>
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</tr>
<tr>
<td>26</td>
<td>0.171806</td>
<td>0.00006</td>
</tr>
<tr>
<td>30</td>
<td>0.184078</td>
<td>0.00002</td>
</tr>
<tr>
<td>31</td>
<td>0.190214</td>
<td>0.00006</td>
</tr>
<tr>
<td>42</td>
<td>0.257709</td>
<td>0.00028</td>
</tr>
<tr>
<td>5A</td>
<td>0.355884</td>
<td>0.00064</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
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<th>PCT</th>
<th>( \Theta(I) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6A</td>
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<td>0.0257529</td>
<td>0.7528313</td>
</tr>
<tr>
<td>75</td>
<td>0.46010426</td>
<td>0.0485516</td>
<td>0.6588277</td>
</tr>
<tr>
<td>76</td>
<td>0.46630115</td>
<td>0.0341753</td>
<td>0.6957203</td>
</tr>
<tr>
<td>77</td>
<td>0.47246608</td>
<td>0.0237300</td>
<td>0.7394726</td>
</tr>
</tbody>
</table>

NSALFR = 203
DEPT OF WATE AT GAUGE SITE = 24.0
AVG = 22.46
AVG = 20.808
SUM = 299
SUM = 293
RATIO = 0.84
RATIO = 0.78
RATIO = 3.579
RATIO = 3.41
SUM = 31019470
BREAKING WAVE HEIGHT MB = 3.61
BREAKING WAVE CELESTY CMB = 12.22
RSHFRQ = 0.0048
RSHFRQ = 0.3742
RSHFRQ = 0.0114
PLP = 125.7133
PLP = 25.6734
PLP = 100.0401

Figure 8. Three examples of output for wave gage pair at Channel Islands Harbor.--Continued
### Example 3

<table>
<thead>
<tr>
<th>GAUGE NO.</th>
<th>MONTH</th>
<th>DAY</th>
<th>TIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>311</td>
<td>7</td>
<td>26</td>
<td>2000</td>
</tr>
<tr>
<td>312</td>
<td>7</td>
<td>26</td>
<td>2000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>I</th>
<th>SIGMA(I)</th>
<th>PCT</th>
<th>THETA(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.006136</td>
<td>0.003930</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.000136</td>
<td>0.000124</td>
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</tr>
<tr>
<td>3</td>
<td>0.001346</td>
<td>0.000134</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.002346</td>
<td>0.000134</td>
<td></td>
</tr>
<tr>
<td>5</td>
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<td></td>
</tr>
<tr>
<td>6</td>
<td>0.004346</td>
<td>0.000134</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 8.** Three examples of output for wave gage pair at Channel Islands Harbor.—Continued
LITERATURE CITED


Walton, Todd L.


A FORTRAN IV computer program (written for the CERC Longshore Sand Transport Research Program and designed to accept data in the CERC magnetic-tape format of record lengths consisting of 4,100 values) is used to analyze wave data collected at Channel Islands Harbor, California. Steps in an analysis of wave data and sample outputs for some wave records from a wave gage pressure sensor pair are given.


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